Los Angeles Southwest College

Mathematics Department

Math 115 – Common Final Exam

Study Guide (solutions)

SPRING 2010

Prepared by: Instr. B.Nash
Dr. L.Saakian

Chapter 1.

1. Simplify: $8+[-2+(-1)]$

   $=8+(-3)$ $=5$

2. Simplify: $-8+[9+(-2)]$

   $=-8+7$ $=-1$

3. Simplify: $[(-9)+(-3)]+12$

   $=(-12)+12$ $=0$

4. Simplify: $[-5+(-9)]+[16+(-21)]$

   $=-14+(-5)$ $=-19$

5. Decide whether the statement is true or false: $-6\le-(-3)$

   $-6\le 3$ *true*

6. Decide whether the statement is true or false: $-|8|>|-9|$

   $-8>9$ *false*

7. Decide whether the statement is true or false: $-4\le-(-5)$

   $-4\le 5$ *true*

8. Decide whether the statement is true or false: $|13-8|\le|7-4|$

   $|5|\le|3|$ $5\le 3$ *false*

9. Perform indicated operation: $\dfrac{5(-2)-3(4)}{-2[3-(-2)]-1}$

   $=\dfrac{-10-12}{-2(3+2)-1}$ $=\dfrac{-10+(-12)}{-2\cdot 5-1}$ $=\dfrac{-22}{-10-1}$

   $=\dfrac{-22}{-10+(-1)}$ $=\dfrac{-22}{-11}$ $=2$

10. Perform indicated operation: $\dfrac{10^2-5^2}{8^2+3^2-(-2)}$

$$= \frac{100-25}{64+9+2} \qquad = \frac{75}{73+2} \qquad = \frac{75}{75} \qquad = 1$$

11. Perform indicated operation: $\frac{(0.6)^2+(0.8)^2}{(-1.2)^2-(-0.56)}$

$$= \frac{0.36+0.64}{1.44+0.56} \qquad = \frac{1}{2}$$

12. For the following word phrase write an expression using x as the variable and simplify. “12 less than the difference between 8 and -5”.

$$= [8-(-5)]-12 \qquad = 13-12 \qquad = 1$$

13. For the following word phrase write an expression using x as the variable and simplify. “The sum of –4 and –10, increased by 12”.

$$= [-4+(-10)]+12 \qquad = -14+12 \qquad = -2$$

14. For the following word phrase write an expression using x as the variable and simplify. “19 less than the difference between 9 and -2”.

$$= [9-(-2)]-19 \qquad = 11-19 \qquad = -8$$

15. For the following word phrase write an expression using x as the variable and simplify. “The sum of 12 and -7,decreased by 14”.

$$= [12+(-7)]-14 \qquad = 5-14 \qquad = -9$$

16. Otis neglects to keep up his checkbook balance. When he finally balanced his account, he found that the balance was -\$23.75, so he deposited \$50.00. What is his new balance?

$$= -\$23.75+\$50.00 \qquad = \$26.25$$

17. Mike O’Hanian owed a friend \$28. He repaid \$13, but than borrowed another \$14. What positive or negative amount represents his present financial status?

$$= -\$28+\$13-\$14 \qquad = -\$15-\$14 \qquad = -\$29$$

18. Peyton Manning of the Indianapolis Colts passed for a gain of 8 yd, was sacked for a loss of 12 yd, and then threw a 42 yd touchdown pass. What positive or negative number represents the total net yardage for the plays?

$$= 8-12+42 \qquad = -4+42 \qquad = 38$$

19. If the temperature drops 7° below its previous level of -3°, what is the new temperature?

$$= -3^\circ - 7^\circ \qquad = -10^\circ$$

20. Evaluate $6x-4z,\ if\ \ x=-5\ \ and\ \ z=-3$

$=6\cdot(-5)-4\cdot(-3)\qquad =-30+12\qquad =-18$

21. Evaluate $3x-4y^2,\ if\ \ x=-2\ \ and\ \ y=4$

$=3\cdot(-2)-4\cdot 4^2\qquad =-6-4\cdot 16\qquad =-6-64\qquad =-70$

22. Evaluate $6x-4z,\ if\ \ x=-5\ \ and\ \ z=-3$

$=6\cdot(-5)-4(-3)\qquad =-30+12\qquad =-18$

23. Evaluate $z^2(3x-8y),\ if\ \ x=-5, y=4,\ and\ \ z=-3$

$=(-3)^2[3\cdot(-5)-8\cdot 4]\qquad =9(-15-32)\qquad =9\cdot(-47)\qquad =-423$

24. Combine like terms: $-5(5y-9)+3(3y+6)$

$=-25y+45+9y+18\qquad =-16y+63$

25. Combine like terms: $-2(3r-4)-(6-r)+2r-5$

$=-6r+8-6+r+2r-5\qquad =-6r+r+2r+8-6-5\qquad =-3r-3$

26. Combine like terms: $2p^2+3p^2-8p^3-6p^3$

$=5p^2-14p^3$

27. Combine like terms: $6-3z-2z-5+z-3z$

$=1-7z$

Chapter 2.

1. Solve the equation and check your solution: $6x+5+7x+3=12x+4$

$13x+8=12x+4\qquad 13x-12x=4-8\qquad x=-4$

$Check.\quad 6(-4)+5+7(-4)+3=12(-4)+4$

$-24+5-28+3=-48+4\qquad -19-28+3=-44\qquad -47+3=-44$
$-44=-44$

2. Solve the equation and check your solution: $4(k-6)-(3k+2)=-5$

$4k-24-3k-2=-5\qquad k-26=-5\qquad k=-5+26\qquad k=21$

$Check.\quad 4(21-6)-(3\cdot 21+2)=-5\quad 4\cdot 15-(63+2)=-5$

$60-65=-5\qquad -5=-5\qquad true$

3. Solve the equation and check your solution: $(5y+6)-(3+4y)=10$

$5y+6-3-4y=10 \qquad y+3=10 \qquad y=10-3 \qquad y=7$

Check. $(5\cdot 7+6)-(3+4\cdot 7)=10 \qquad (35+6)-(3+28)=10$

$41-31=10 \qquad 10=10 \qquad$ *true*

4. Solve the equation and check your solution: $2(2-3r)=-5(r-3)$

$4-6r=-5r+15 \qquad -6r+5r=15-4 \qquad -r=11 \qquad r=-11$

Check. $2\ \ 2-3(-11)\ =-5(-11-3) \qquad 2(2+33)=-5\cdot(-14)$

$2\cdot 35=70 \qquad 70=70 \qquad$ *true*

5. Solve the equation and check your solution: $4x+3x=21$

$7x=21 \qquad x=3$

Check. $4\cdot 3+3\cdot 3=21 \qquad 12+9=21 \qquad 21=21 \qquad$ *true*

6. Solve the equation and check your solution: $5m+6m-2m=63$

$9m=63 \qquad m=7$

Check. $5\cdot 7+6\cdot 7-2\cdot 7=63 \qquad 35+42-14=63$

$77-14=63 \qquad 63=63 \qquad$ *true*

7. Solve the equation and check your solution: $11r-5r+6r=168$

$12r=168 \qquad r=14$

Check. $11\cdot 14-5\cdot 14+6\cdot 14=168 \qquad 154-70+84=168$

$84+84=168 \qquad 168=168 \qquad$ *true*

8. Solve the equation and check your solution: $9p-13p=24$

$-4p=24 \qquad p=-6$

Check. $9\cdot(-6)-13(-6)=24 \qquad -54+78=24 \qquad 24=24 \qquad$ *true*

9. Solve the equation and check your solution: $5(2m+3)-4m=8m+27$

$10m+15-4m=8m+27 \qquad 6m+15=8m+27$

$6m-8m=27-15 \qquad -2m=12 \qquad m=-6$

Check. $5\ \ 2(-6)+3\ \ -4(-6)=8(-6)+27 \qquad 5(-12+3)+24=-48+27$

$5\cdot(-9)+24=-21$

$-45+24=-21 \qquad -21=-21 \qquad$ *true*

10. Solve the equation and check your solution: $6(4x-1)=12(2x+3)$

$24x-6=24x+36 \qquad 24x-24x=36+6 \qquad 0=42 \qquad N/S$

11. Solve the equation and check your solution: $3(2x-4)=6(x-2)$

$6x-12=6x-12 \qquad 6x-6x=-12+12 \qquad 0=0 \qquad \textit{all real numbers}$

12. Solve the equation and check your solution: $7r-5r+2=5r-r$

$2r+2=4r \qquad 2r-4r=0-2 \qquad -2r=-2 \qquad r=1$

$Check. \quad 7\cdot 1-5\cdot 1+2=5\cdot 1-1 \qquad 7-5+2=5-1 \qquad 4=4$

13. The sum of three times a number and 7 more than the number is the same as the difference between –11 and twice the number. What is the number?

$3x+(x+7)=-11-2x \qquad 3x+x+7=-11-2x \qquad 4x+7=-11-2x$

$4x+2x=-11-7 \qquad 6x=-18 \qquad x=-3$

14. During the 109th Congress (2005-2006), the U.S. Senate had a total of 99 Democrats and Republicans. There were 11 more Republicans than Democrats. How many Democrats and Republicans were there in the Senate?

$Let\ x\ is\ \#\ of\ democrats \qquad \#\ of\ republicans\ is\ x+11$

$x+(x+11)=99 \qquad x+x+11=99 \qquad 2x+11-11=99-11$

$2x=88 \qquad x=44 \qquad \#\ of\ democrats\ is\ 44$

$\#\ of\ republicans\ is\ 44+11=55$

15. In one day, a store sold $\frac{8}{5}$ as many DVDs as CDs. The total number of DVDs and CDs sold that day was 273. How many DVDs were sold?

$Let\ x\ is\ \#\ of\ CDs\ sold \qquad \#\ of\ DVDs\ is\ x\cdot\frac{8}{5}$

$x+x\cdot\frac{8}{5}=273 \qquad \frac{13}{5}x=273 \qquad \frac{5}{13}\cdot\frac{13}{5}x=273\cdot\frac{5}{13} \qquad x=105$

$\#of\ CDs\ is\ 105 \qquad \#of\ DVDs\ is\ 105\cdot\frac{8}{5}=168$

16. In her job as a mathematics textbook editor, Lauren Morse works 7.5 hr a day. She spent a recent day making telephone calls, writing e-mails, and attending meetings. On that day, she spent twice as much time attending meetings as making telephone calls and spent 0.5 hr longer writing e-mails than making telephone calls. How many hours did she spend on each task?

$Let\ x\ is\ a\ time\ she\ spent\ making\ telephone\ calls.$

$The\ time\ attending\ meetings\ is\ 2x \qquad The\ time\ writing\ e-mails\ is\ x+0.5$

$x+2x+x+0.5=7.5 \qquad 4x=7.5-0.5 \qquad 4x=7$

$x=\frac{7}{4} \qquad x=1.75$

$telephone\ calls: 1.75hr,\ e-mails: 2.25hr, meetings: 3.5hr$

17. The supplement of an angle measures 10 times the measure of its complement. What is the measure of the angle?

$Let\ x\ \ is\ a\ value\ of\ \ the\ angle. The\ value\ of\ \ the\ supplement\ angle$

$is\,180 - x\ and\ value\ of\ \ complement\ angle\ is\ 90 - x.$

$180 - x = 10(90 - x) \quad 180 - x = 900 - 10x \quad 9x = 720 \quad x = 80^{o}$

18. Find two consecutive odd integers such that when the lesser is added to twice the greater, the result is 24 more than the greater integer.

$Let\ x\ \ is\ a\ value\ of\ \ the\ lesser\ odd\ integer. The\ value\ of\ \ the\ greater$

$odd\ integer\ is\ x + 2.$

$2(x+2) + x = (x+2) + 24 \quad 2x + 4 + x = x + 2 + 24$

$3x + 4 = x + 26 \quad 2x = 22 \quad x = 11$

$The\ numbers\ are\ 11\ and\ 13$

19. The perimeter of a certain rectangle is 16 times the width. The length is 12 cm more than the width. Find the width of the rectangle.

$Let\ x\ \ is\ a\ value\ of\ \ the\ width. The\ value\ of\ \ the\ length$

$is\ x + 12. The\ \ perimeter\ is\ equals\ 2(width\ \ plus\ length)$

$2\ \ x + (x+12)\ \ = 16x \quad 2(2x+12) = 16x \quad 4x + 24 = 16x$

$12x = 24 \quad x = 2$

$The\ width\ is\ 2$

20. Two trains are 390 mi apart. They start at the same time and travel toward one another, meeting 3 hr later. If the speed of one train is 30 mph more than the speed of the other train, find the speed of each train.

$Let\ x\ \ is\ the\ speed\ of\ \ the\ \ first\ train. The\ speed\ of\ \ the\ second$

$train\ is\ x + 30. The\ sum\ of\ \ the\ distance\ made\ by\ trains\ in\ 3\ hr\ is\ 390\ mi$

$3x + 3(x+30) = 390 \quad 3x + 3x + 90 = 390 \quad 6x = 300 \quad x = 50$

$The\ speeds\ of\ \ the\ trains\ are\ 50mph\ and\ 80mph$

21. The perimeter of a triangle is 96 m. One side is twice as long as another and the third side is 30 m long. What is the length of the longest side?

$Let\ x\ \ is\ the\ length\ of\ \ the\ second\ \ side. The\ length\ of\ \ the\ \ first$

$side\ is\ 2x. The\ sum\ of\ \ all\ three\ sides\ is\ 96\ m$

$2x + x + 30 = 96 \quad 3x = 66 \quad x = 22$

$The\ length\ of\ \ the\ longest\ side\ is\ 44\ m$

22. The perimeter of a basketball court is 288 ft. The width of the court is 44 ft less than the length. What are the dimensions of the court?

$Let\ x\ \ is\ the\ length\ of\ \ the\ court. The\ width\ is\ x-44.$

$2[x+(x-44)]=288 \quad 2(2x-44)=288 \qquad 4x-88=288$

$4x=376 \qquad x=94$

$The\ length\ of\ \ the\ court\ is\ 94\ \ ft\ and\ the\ width\ is\ 50\ \ ft$

23. Solve the formula $d=rt \quad for\ t$

$\frac{d}{r}=\frac{rt}{r} \qquad t=\frac{d}{r}$

24. Solve the formula $P=2L+2W \quad for\ W$

$P-2L=2W \quad W=\frac{P-2L}{2}$

25. Solve the formula $M=C(1+r) \quad for\ r$

$1+r=\frac{M}{C} \qquad r=\frac{M}{C}-1$

26. Solve the formula $C=\frac{5}{9}(F-32) \quad for\ F$

$F-32=C\cdot\frac{9}{5} \qquad F=\frac{9}{5}C+32$

27. Solve the formula $A=\frac{1}{2}h(b+B) \quad for\ h$

$2A=h(b+B) \qquad h=\frac{2A}{b+B}$

28. Solve the equation $\frac{x}{6}=\frac{18}{4}$

$4x=18\cdot 6 \qquad 4x=108 \qquad x=27$

29. Solve the equation $\frac{3y-2}{5}=\frac{6y-5}{11}$

$11(3y-2)=5(6y-5) \qquad 33y-22=30y-25 \qquad 33y-30y=-25+22$

$3y=-3 \qquad y=-1$

30. If 6 gal of premium unleaded gasoline costs \$19.56, how much would it cost to completely fill a 15-gal tank?

$Let\ costs\ of\ \ the\ tank\ is\ x \qquad The\ \ proportion\ is\ \ \frac{6}{15}=\frac{19.56}{x}$

$6x=15\cdot 19.56 \qquad x=\frac{15\cdot 19.56}{6} \qquad x=48.90$

The costs to fill a tank is \$48.90

31. The distance between Singapore and Tokyo is 3300 mi. On a certain wall map, this is represented by 11 in. The actual distance between Mexico City and Cairo is 7700 mi. How far apart are they on the same map?

$\textit{Let distance on the map is } x$ $\quad$ $\textit{The proportion is } \frac{3300}{7700}=\frac{11}{x}$

$3300x = 11\cdot 7700$ $\quad x=\frac{11\cdot 7700}{3300}$ $\quad x=\frac{77}{3}$ $\quad x=25\frac{2}{3}$

The distance between Singapore and Tokyo on the map is $25\frac{2}{3}$ in.

32. Solve the inequality $6x+3+x<2+4x+4$

$7x+3<4x+6$ $\quad 7x-4x<6-3$ $\quad 3x<3$ $\quad x<1$ $\quad (-\infty,1)$

33. Solve the inequality $5(x+3)-6x\le 3(2x+1)-4x$

$5x+15-6x\le 6x+3-4x$ $\quad -x+15\le 2x+3$ $\quad -x-2x\le 3-15$

$-3x\le -12$ $\quad x\ge 4$ $\quad [4,\infty)$

34. Solve the inequality $-5\le 2x-3\le 9$

$-5+3\le 2x\le 9+3$ $\quad -2\le 2x\le 12$ $\quad -1\le x\le 6$ $\quad [-1,6]$

35. Solve the inequality $-1\le 1-5q\le 16$

$-1-1\le -5q\le 16-1$ $\quad -2\le -5q\le 15$ $\quad \frac{2}{5}\ge q\ge -3$ $\quad \left[-3,\frac{2}{5}\right]$

Chapter 3.

1. Find the x-intercept and y-intercept: $2x-3y=24$

$x=0 \quad -3y=24 \quad y=-8$

$y=0 \quad 2x=24 \quad x=12$

$(12,0);(0,-8)$

2. Find the x-intercept and y-intercept: $5x-2y=20$

$x=0 \quad -2y=20 \quad y=-10$

$y=0 \quad 5x=20 \quad x=4$

$(4,0);(0,-10)$

3. Find the x-intercept and y-intercept: $y+1.5=0$

$y = -1.5$

$none; (0, -1.5)$

4. Find the x-intercept and y-intercept: $x - 4 = 0$

$x = 4$

$(4, 0); none$

5. Find the slope of the line through pair of points: $(4, -1)$ *and* $(-2, -8)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{-8 - (-1)}{-2 - 4}$ $\quad m = \frac{-8 + 1}{-2 + (-4)}$ $\quad m = \frac{-7}{-6}$ $\quad m = \frac{7}{6}$

6. Find the slope of the line through pair of points: $(-8, 0)$ *and* $(0, -5)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{-5 - 0}{0 - (-8)}$ $\quad m = \frac{-5}{8}$ $\quad m = -\frac{5}{8}$

7. Find the slope of the line through pair of points: $(-8, 6)$ *and* $(-8, -1)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{-1 - 6}{-8 - (-8)}$ $\quad m = \frac{-1 + (-6)}{-8 - (-8)}$ $\quad m = \frac{-7}{0}$

$m = underfind$

8. Find the slope of the line through pair of points: $(6, -5)$ *and* $(-12, -5)$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \frac{-5 - (-5)}{-12 - 6}$ $\quad m = \frac{0}{-18}$ $\quad m = 0$

9. For the pair of equation, give the slopes of the lines and then determine whether the two lines are parallel, perpendicular, or neither parallel nor perpendicular.

$$\begin{cases} 2x + 5y = 4 \\ 4x + 10y = 1 \end{cases}$$

$5y = -2x + 4$ $\quad y = -\frac{2}{5}x + \frac{4}{5}$ $\quad m_1 = -\frac{2}{5}$

$10y = -4x + 1$ $\quad y = -\frac{2}{5}x + \frac{1}{10}$ $\quad m_2 = -\frac{2}{5}$

$m_1 = m_2 - two\ lines\ are\ parallel$

10. For the pair of equation, give the slopes of the lines and then determine whether the two lines are parallel, perpendicular, or neither parallel nor perpendicular.

$$\begin{cases} 3x - 2y = 6 \\ 2x + 3y = 3 \end{cases}$$

$$-2y=-3x+6 \quad y=\frac{3}{2}x-3 \quad m_1=\frac{3}{2}$$

$$3y=-2x+3 \quad y=-\frac{2}{3}x+1 \quad m_2=-\frac{2}{3}$$

$$m_1 \cdot m_2=-1 - \textit{two lines are perpendicular}$$

11. For the pair of equation, give the slopes of the lines and then determine whether the two lines are parallel, perpendicular, or neither parallel nor perpendicular.

$$\begin{cases} 8x-9y=6 \\ 8x+6y=-5 \end{cases}$$

$$-9y=-8x+6 \quad y=\frac{8}{9}x-\frac{2}{3} \quad m_1=\frac{8}{9}$$

$$6y=-8x-5 \quad y=-\frac{4}{3}x-\frac{5}{6} \quad m_2=-\frac{4}{3}$$

$$m_1 \neq m_2 \textit{ and } m_1 \cdot m_2 \neq -1 - \textit{two lines are neither parallel nor perpendicular}$$

12. For the pair of equation, give the slopes of the lines and then determine whether the two lines are parallel, perpendicular, or neither parallel nor perpendicular.

$$\begin{cases} 5x-y=1 \\ x-5y=-10 \end{cases}$$

$$-y=-5x+1 \quad y=5x-1 \quad m_1=5$$

$$-5y=-x-10 \quad y=-\frac{1}{5}x+2 \quad m_2=-\frac{1}{5}$$

$$m_1 \cdot m_2=-1 - \textit{two lines are perpendicular}$$

13. Write an equation for the line passing through the given point and having the given slope. Give the final answer in slope-intercept form: $(4,1),\ m=2$

$$y-y_1=m(x-x_1)$$

$$y-1=2(x-4) \quad y-1=2x-8 \quad y=2x-7$$

14. Write an equation for the line passing through the given point and having the given slope. Give the final answer in slope-intercept form: $(-2,5),\ m=\frac{2}{3}$

$$y-y_1=m(x-x_1)$$

$$y-5=\frac{2}{3}[x-(-2)] \quad y-5=\frac{2}{3}x+\frac{4}{3} \quad y=\frac{2}{3}x+\frac{19}{3}$$

15. Write an equation for the line passing through the given point and having the given slope. Give the final answer in slope-intercept form: $(-1,3),\ m=-4$

$$y-y_1=m(x-x_1)$$

$y-3=-4[x-(-1)] \quad y-3=-4x-4 \quad y=-4x-1$

16. Write an equation for the line passing through the given point and having the given slope. Give the final answer in slope-intercept form: $(2,7),\ m=3$

$$y-y_1=m(x-x_1)$$

$$y-7=3(x-2) \quad y-7=3x-6 \quad y=3x+1$$

17. Write an equation for the line passing through the given pair of points. Give the final answer in slope-intercept form. $(8,5)$ *and* $(9,6)$

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$m=\frac{6-5}{9-8} \quad m=\frac{1}{1} \quad m=1$$

$$y-y_1=m(x-x_1)$$

$$y-5=1(x-8) \quad y-5=x-8 \quad y=x-3$$

18. Write an equation for the line passing through the given pair of points. Give the final answer in slope-intercept form. $(4,10)$ *and* $(6,12)$

$$m=\frac{y_2-y_1}{x_2-x_1}$$

$$m=\frac{12-10}{6-4} \quad m=\frac{2}{2} \quad m=1$$

$$y-y_1=m(x-x_1)$$

$$y-10=1(x-4) \quad y-10=x-4 \quad y=x+6$$

19. Write an equation for the line passing through the given pair of points. Give the final answer in slope-intercept form. $(-2,-1)$ *and* $(3,-4)$

$$m=\frac{y_2-y_1}{x_2-x_1} \quad m=\frac{-4-(-1)}{3-(-2)} \quad m=\frac{-3}{5} \quad m=-\frac{3}{5}$$

$$y-y_1=m(x-x_1)$$

$$y-(-1)=-\frac{3}{5}[x-(-2)] \quad y+1=-\frac{3}{5}x-\frac{6}{5} \quad y=-\frac{3}{5}x-\frac{11}{5}$$

20. Write an equation for the line passing through the given pair of points. Give the final answer in slope-intercept form. $(-4,0)$ *and* $(0,2)$

$$m=\frac{y_2-y_1}{x_2-x_1} \quad m=\frac{2-0}{0-(-4)} \quad m=\frac{2}{4} \quad m=\frac{1}{2}$$

$$y-y_1=m(x-x_1)$$

$y-0=\frac{1}{2}[x-(-4)] \quad y=\frac{1}{2}x+2$

21. Graph the linear inequality: $3x+5y>9$

To graph the boundary, which is the line $3x+5y=9$, find its intercepts:

$3x+5y=9 \qquad 3x+5y=9$

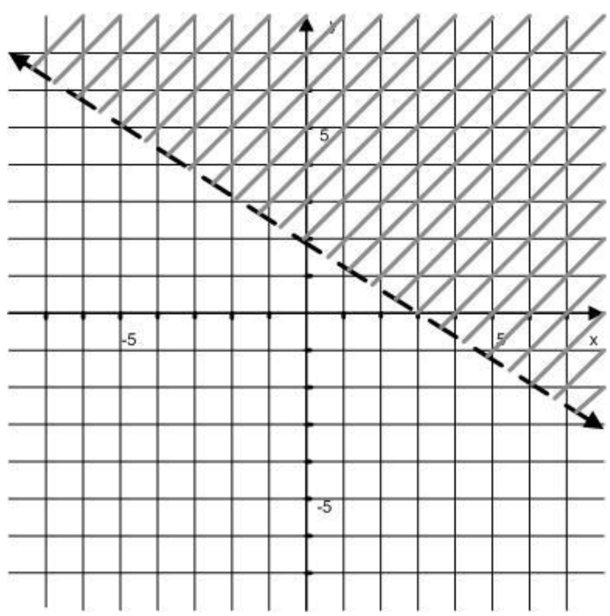
$let \quad y=0 \qquad let \quad x=0$

$3x+5\cdot 0=9 \qquad 3\cdot 0+5y=9$

$3x=9 \qquad 5y=9$

$x=3 \qquad y=\frac{9}{5}$

The $x-intercept$ is $(3,0)$ and $y-intercept$ is $\left(0,\frac{9}{5}\right)$.draw a dashed line through these points. In order to determine which side of the line should be shaded, use $(0,0)$ as a test point. Substituting 0 for x and y will result in the inequality $0>9$, which is $false$. Shade the region *not* containing the origin. The dashed line shows that the boundary is not part of the graph.



22. Graph linear inequality: $2x-3y>-6$

To graph the boundary, which is the line $2x-3y=-6$, find its intercepts:

$2x-3y=-6 \qquad 2x-3y=-6$

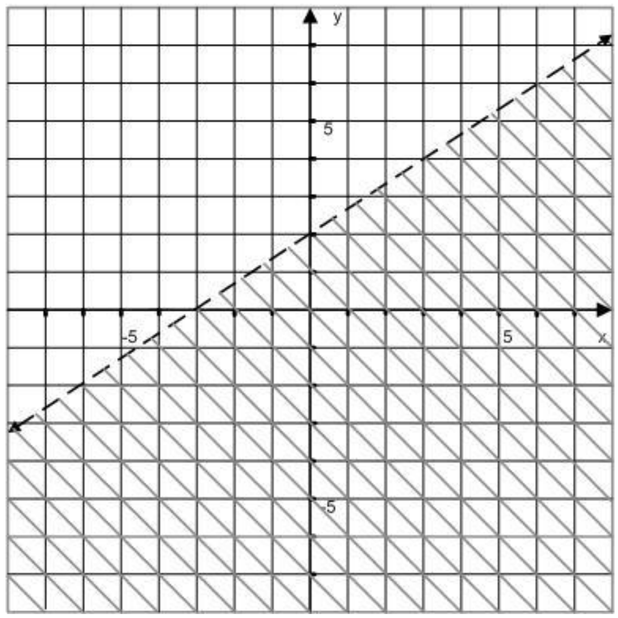
$let \quad y=0 \qquad let \quad x=0$

$2x-3\cdot 0=-6 \qquad 2\cdot 0-3y=-6$

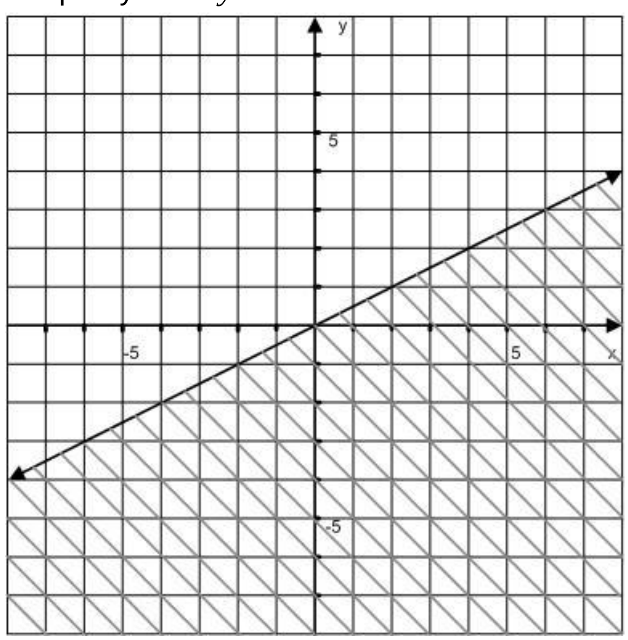
$2x = -6 \qquad\qquad -3y = -6$

$x = -3 \qquad\qquad y = 2$

The $x-intercept$ is $(-3,0)$ and $y-intercept$ is $(0,2)$ .draw a dashed line through these points. In order to determine which side of the line should be shaded, use $(0,0)$ as a test point. Substituting 0 for x and y will result in the inequality $0 > -6$, which is $true$. Shade the region containing the origin. The dashed line shows that the boundary is not part of the graph.



23. Graph linear inequality: $x - 2y \geq 0$



The equation of the boundary is $x-2y=0$. This line goes through the origin, so both intercepts are $(0,0)$. Second point on this line is $(2,1)$. Draw a solid line through $(0,0)$ and $(2,1)$.Because $(0,0)$ lies on the boundary, we must choose another point as the test point. Using $(0,3)$ results in the inequality $-6\ge 0$, which is *false*. Shade the region *not* containing the test point. The solid line shows that the boundary is part of the graph.

Chapter 4.

1. Decide whether the given ordered pair is a solution of the given system.

$(3,4)$

$$\begin{cases}4x-2y=4\\5x+y=19\end{cases}$$

$4(3)-2(4)=4 \quad 12-8=4 \quad 4=4 \quad true$

$5(3)+4=19 \quad 15+4=19 \quad 19=19 \quad true$

The ordered pair $(3,4)$ *is a solution of the given system*

2. Decide whether the given ordered pair is a solution of the given system.

$(-5,2)$

$$\begin{cases}x-4y=-13\\2x+3y=4\end{cases}$$

$(-5)-4(2)=-13 \quad -5-8=-13 \quad -13=-13 \quad true$

$2(-5)+3(2)=4 \quad -10+6=4 \quad -4=4 \quad false$

The ordered pair $(-5,2)$ *is not a solution of the given system*

3. Solve the system by graphing:

$$\begin{cases}x+y=4\\2x-y=5\end{cases}$$

To graph the equations, find the intercepts.

$x+y=4; \quad let\ y=0; then\ x=4$

$\quad\quad let\ x=0; then\ y=4$

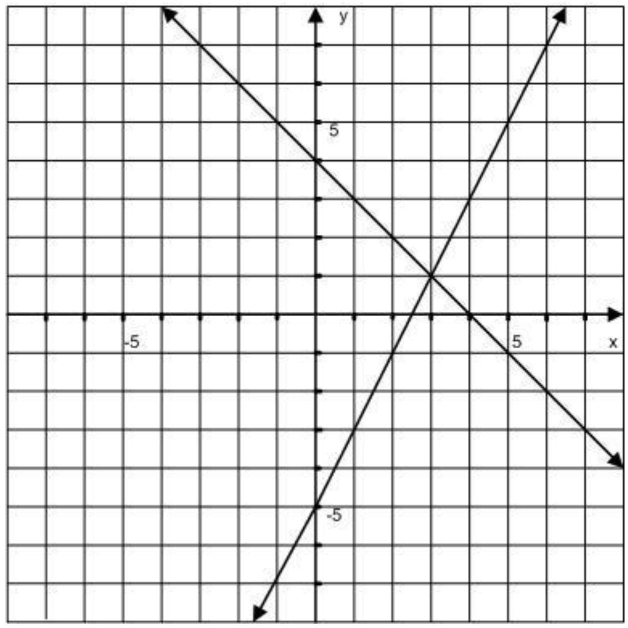
Plot the intercepts, $(4,0)$ *and* $(0,4)$, and draw the line trough them

$2x-y=5; \quad let\ y=0; then\ x=\frac{5}{2}$

$\quad\quad let\ x=0; then\ y=-5$

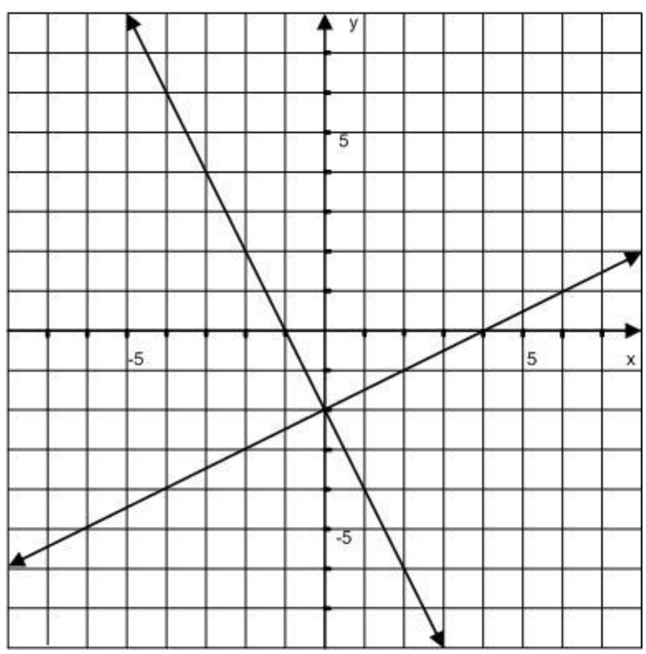
Plot the intercepts, $(\frac{5}{2},0)$ *and* $(0,-5)$, and draw the line trough them

It appears that the lines intersect at the point $(3,1)$. Check this by substituting $3\ \textit{for}\ x\ \textit{and}\ 1\ \textit{for}\ y$ in both equations. Since $(3,1)$ satisfies both equations, the solution set of this system is $\{(3,1)\}$



4. Solve the system by graphing:

$$\begin{cases} x - 2y = 4 \\ 2x + y = -2 \end{cases}$$



To graph the equations, find the intercepts.

$x-2y=4; \quad let\ y=0; then\ x=4$

$let\ x=0; then\ y=-2$

Plot the intercepts, $(4,0) \quad and \quad (0,-2)$, and draw the line trough them

$2x+y=-2; \quad let\ y=0; then\ x=-1$

$let\ x=0; then\ y=-2$

Plot the intercepts, $(-1,0) \quad and \quad (0,-2)$, and draw the line trough them

It appears that the lines intersect at the point $(0,-2)$. Check this by substituting $0\ for\ x\ and\ -2\ for\ y$ in both equations. Since $(0,-2)$ satisfies both equations, the solution set of this system is $\{(0,-2)\}$

5. Solve the system by the substitution method.

$$\begin{cases} 3x-2y=19 \\ x+y=8 \end{cases}$$

$x+y=8 \quad x=-y+8$

$3x-2y=19 \quad 3(-y+8)-2y=19 \quad -5y+24=19 \quad -5y=-5 \quad y=1$

$x=-y+8 \quad x=-1+8 \quad x=7$

$\{(7,1)\}$

6. Solve the system by the substitution method.

$$\begin{cases} 2x+y=0 \\ 4x-2y=2 \end{cases}$$

$2x+y=0 \quad y=-2x$

$4x-2(-2x)=2 \quad 4x+4x=2 \quad x=\frac{1}{4}$

$y=-2x \quad y=-2\cdot\frac{1}{4} \quad y=-\frac{1}{2}$

$\left\{\left(\frac{1}{4},-\frac{1}{2}\right)\right\}$

7. Solve the system by the substitution method.

$$\begin{cases} x+y=12 \\ y=3x \end{cases}$$

$x+y=12 \quad x+3x=12 \quad 4x=12 \quad x=3$

$y=3x \quad y=3\cdot 3 \quad y=9$

$\{(3,9)\}$

8. Solve the system by the substitution method.

$$\begin{cases} 2y = 14 - 6x \\ 3x + y = 7 \end{cases}$$

$y = 7 - 3x$

$3x + y = 7 \quad 3x + (7 - 3x) = 7 \quad 3x + 7 - 3x = 7 \quad 7 = 7 \quad \textit{true}$

$\{(x, y) \mid 3x + y = 7\}$

9. Solve the system by the elimination method.

$$\begin{cases} 2x + y = -5 \\ x - y = 2 \end{cases}$$

$$\begin{array}{l} 2x + y = -5 \\ x \;\; - y = 2 \\ \hline 3x \qquad = -3 \end{array} \quad x = -1$$

$x - y = 2 \quad -1 - y = 2 \quad -y = 3 \quad y = -3$

$\{(-1, -3)\}$

10. Solve the system by the elimination method.

$$\begin{cases} 2x - y = 12 \\ 3x + 2y = -3 \end{cases}$$

$2 \cdot | \, 2x - y = 12$

$4x - 2y = 24$

$$\begin{array}{l} 3x + 2y = -3 \\ \hline 7x \qquad = 21 \end{array} \quad x = 3$$

$2x - y = 12 \quad 2 \cdot 3 - y = 12 \quad 6 - y = 12 \quad -y = 6 \quad y = -6$

$\{(3, -6)\}$

11. Solve the system by the elimination method.

$$\begin{cases} 3x = 3 + 2y \\ -\frac{4}{3}x + y = \frac{1}{3} \end{cases}$$

$$\begin{cases} 3x = 3 + 2y \\ 3 \cdot | -\frac{4}{3}x + y = \frac{1}{3} \end{cases}$$

$$\begin{cases} 3 \cdot | \, 3x - 2y = 3 \\ 2 \cdot | -4x + 3y = 1 \end{cases}$$

$$9x - 6y = 9$$
$$-8x + 6y = 2$$
$$x \qquad = 11$$
$$3 \cdot 11 - 2y = 3 \quad -2y = -30 \quad y = 15$$
$$\{(11, 15)\}$$

12. Solve the system by the elimination method.

$$\begin{cases} 5x - 2y = 3 \\ 10x - 4y = 5 \end{cases}$$

$$\begin{cases} -2 \cdot | \, 5x - 2y = 3 \\ 10x - 4y = 5 \end{cases}$$

$$-10x + 4y = -6$$
$$10x - 4y = 5$$
$$0 \qquad = -1$$
$$false \quad n/s$$
$$\varnothing$$

13. Bill Kunz went to the post office to stock up on stamps. He spent $19.44 on 56 stamps, made up of a combination of 39-cent and 24-cent stamps. How many stamps of each denomination did he buy?

$$let\ x\ is\ amount\ of\ 39-cent\ stamps\ and\ y\ is\ amount\ of\ 24-cent\ stamps$$

$$\begin{cases} x + y = 56 \\ 39x + 24y = 1944 \end{cases}$$

$$\begin{cases} -24 \cdot | \, x + y = 56 \\ 39x + 24y = 1944 \end{cases}$$

$$-24x - 24y = -1344$$
$$39x + 24y = 1944$$
$$15x \qquad = 600 \quad x = 40$$
$$x + y = 56 \quad 40 + y = 56 \quad y = 16$$
$$39-cent: 40;\ 24-cent: 16$$

14. A 40% dye solution is to be mixed with a 70% dye solution to get 120 L of a 50% solution. How many liters of the 40% and 70% solutions will be needed?

$$let\ x\ is\ volume\ of\ the\ 40\%\ dye\ solution\ and\ y\ is\ volume\ of\ the\ 70\%\ dye\ solution$$

$$\begin{cases} x+y=120 \\ 10\cdot|\,0.4x+0.7y=0.5\cdot 120 \end{cases}$$

$$\begin{cases} -4\cdot|\,x+y=120 \\ 4x+7y=600 \end{cases}$$

$$\begin{aligned} -4x-4y&=-480 \\ 4x+7y&=600 \\ \hline 3y&=120 \quad y=40 \end{aligned}$$

$x+y=120 \quad x+40=120 \quad x=80$

$a\ 40\%\ dye\ solution: 80L;\ a\ 70\%\ dye\ solution: 40L$

15. Two trains start from towns 495 mi apart and travel toward each other on parallel tracks. They pass each other 4.5 hr later. If one train travels 10 mph faster than the other, find the speed of each train.

$let\ x\ is\ the\ speed\ of\ the\ first\ train\ and\ y\ is\ the\ speed\ of\ the\ second\ train$

$$\begin{cases} x-y=10 \\ 4.5x+4.5y=495 \end{cases}$$

$$\begin{cases} -4.5\cdot|\,x-y=10 \\ 4.5x+4.5y=495 \end{cases}$$

$$\begin{aligned} -4.5x+4.5y&=-45 \\ 4.5x+4.5y&=495 \\ \hline 9y&=450 \quad y=50 \end{aligned}$$

$x-y=10 \quad x-50=10 \quad x=60 \quad -y=6 \quad y=-6$

$first\ train: 60\ mph;\ second\ train: 50\ mph$

16. If a plane can travel 440 mph into the wind and 500 mph with the wind, find the speed of the wind and the speed of the plane in still air.

$let\ x\ is\ the\ speed\ of\ the\ plane\ and\ y\ is\ the\ speed\ of\ the\ wind$

$$\begin{cases} x-y=440 \\ x+y=500 \end{cases}$$

$$\begin{aligned} x-y&=440 \\ x+y&=500 \\ \hline 2x\quad&=940 \quad x=470 \end{aligned}$$

$x-y=10 \quad 470-y=440 \quad -y=-30 \quad y=30$

$plain: 470\ mph;\ wind: 30\ mph$

7. Nancy Johnson invested $18,000. Part of it was invested at 3% annual simple interest, and the rest was invested at 4%. Her interest income for the first year was $650. How much did she invest at each rate?

$$let\ x\ is\ the\ \ first\ investment, the\ second\ is\ 18000-x$$

$$x\cdot 3\% + (18000-x)\cdot 4\% = 650$$

$$\frac{3x}{100}+\frac{4(18000-x)}{100}=650$$

$$3x+72000-4x=65000$$

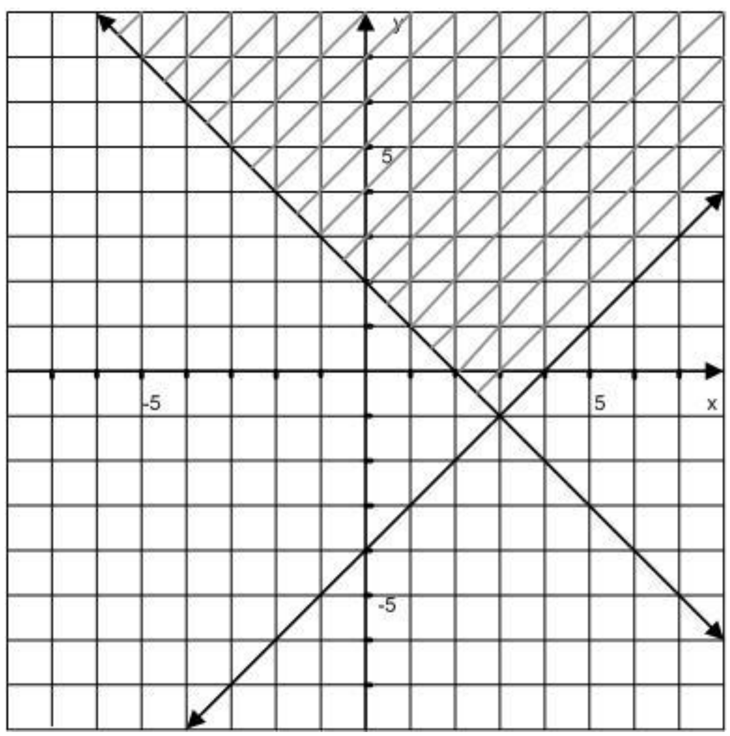
$$-x=-7000$$

$$x=7000$$

She invested \$7,000 *at* 3% *and* \$11,000 *at* 4%

18. Graph the solution set of the system of linear inequalities.

$$x+y\geq 2$$
$$x-y\leq 4$$



Graph $x+y=2$ as a solid line through its intercepts, $(2,0)$ *and* $(0,2)$. Using $(0,0)$ as a test point will result in the *false* statement $0\geq 2$,so shade the region *not* containing the origin.

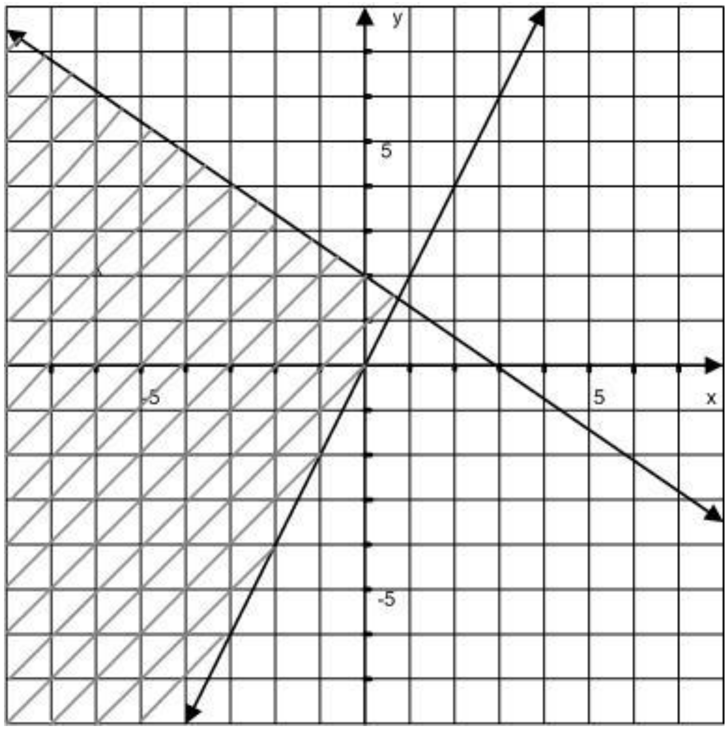
Graph $x-y=4$ as a solid line through its intercepts, $(4,0)$ *and* $(0,4)$. Using $(0,0)$ as a test point will result in the *true* statement $0\leq 4$,so shade the region containing the origin.

The solution set of this system is the intersection of the two shaded regions, and includes the portions of the two lines that bound this region.

19. Graph the solution set of the system of linear inequalities.

$y \geq 2x$

$2x + 3y \leq 6$



Graph $y = 2x$ as a solid line through its intercepts, $(0,0)$ *and* $(1,2)$. This line goes through the origin, so a different test point must be used. Choosing $(-4,0)$ as a test point will result in the *true* statement $0 \geq -8$, so shade the region containing $(-4,0)$.

Graph $2x + 3y = 6$ as a solid line through its intercepts, $(3,0)$ *and* $(0,2)$. Using $(0,0)$ as a test point will result in the *true* statement $0 \leq 6$, so shade the region containing the origin.

The solution set of this system is the intersection of the two shaded regions, and includes the portions of the two lines that bound this region.

Chapter 5.

1. Simplify: $(3x^4y^2z)^3(yz^4)^5$

$= 3^3x^{12}y^6z^3y^5z^{20} \qquad = 27x^{12}y^{11}z^{23}$

2. Simplify: $\left(\dfrac{5a^2b^5}{c^6}\right)^3$

$$=\frac{5^3a^6b^{15}}{c^{18}} \qquad =\frac{125a^6b^{15}}{c^{18}}$$

3. Simplify: $\left(\frac{6x^3y^9}{z^5}\right)^4$

$$=\frac{6^4x^{12}y^{36}}{z^{20}} \qquad =\frac{1296a^6b^{15}}{c^{18}}$$

4. Simplify: $(-r^4s)^2(-r^2s^3)^5$

$$=(-1)^2r^8s^2(-1)^5r^{10}s^{15} \qquad =(-1)^5r^{18}s^{17} \qquad =-r^{18}s^{17}$$

5. Simplify. Use only positive exponents: $(6x^{-5}z^3)^{-3}$

$$=(6)^{-3}x^{15}z^{-9} \qquad =\frac{x^{15}}{(6)^3z^9} \qquad =\frac{x^{15}}{108z^9}$$

6. Simplify. Use only positive exponents: $\frac{(2xy^{-1})^3}{2^3x^{-3}y^2}$

$$=\frac{2^3x^3y^{-3}}{2^3x^{-3}y^2} \qquad =2^0x^6y^{-5} \qquad =\frac{x^6}{y^5}$$

7. Simplify. Use only positive exponents: $(-r^4s)^2(-r^2s^3)^5$

$$=(-1)^2r^8s^2(-1)^5r^{10}s^{15} \qquad =(-1)^7r^{18}s^{17} \qquad =-r^{18}s^{17}$$

8. Simplify. Use only positive exponents: $\left(\frac{mn^{-2}p}{m^2np^4}\right)^{-2}\left(\frac{mn^{-2}p}{m^2np^4}\right)^3$

$$=(m^{-1}n^{-3}p^{-3})^1 \qquad =\frac{1}{m^1n^3p^3}$$

9. Perform the operation: $(9a^4-3a^2+2)+(4a^4-4a^2+2)+(-12a^4+6a^2-3)$

$$\begin{array}{l} 9a^4-3a^2+2 \\ 4a^4-4a^2+2 \\ -12a^4+6a^2-3 \\ \hline a^4 \quad -a^2 \quad +1 \end{array}$$

10. Perform the operation: $(8m^2-7m)-(3m^2+7m-6)$

$$\begin{array}{l} \ \ 8m^2-7m \\ -3m^2-7m+6 \\ \hline 5m^2-14m+6 \end{array}$$

11. Perform the operation: $(6b+3c)+(-2b-8c)$

$$\begin{array}{r} 6b+3c \\ -2b-8c \\ \hline 4b-5c \end{array}$$

12. Perform the operation: $(4x+2xy-3)-(-2x+3xy+4)$

$$\begin{array}{r} 4x+2xy-3 \\ +2x-3xy-4 \\ \hline 6x-1xy-7 \end{array}$$

13. Find the product: $(m+7)(m+5)$

$=m^2+5m+7m+35$ $\quad\quad =m^2+12m+35$

14. Find the product: $(2m-3n)(m+5n)$

$=2m^2+10mn-3mn-15n^2$ $\quad\quad =2m^2+7mn-15n^2$

15. Find the product: $3p^3(2p^2+5p)(p^3+2p+1)$

$=(6p^5+15p^4)(p^3+2p+1)$ $\quad\quad =6p^8+12p^6+6p^5+15p^7+30p^5+15p^4$

$=6p^8+15p^7+12p^6+36p^5+15p^4$

16. Find the product: $(2a+1)^3$

$=(2a+1)(2a+1)(2a+1)$ $\quad\quad =(4a^2+4a+1)(2a+1)$

$=8a^3+4a^2+8a^2+4a+2a+1$ $\quad\quad =8a^3+12a^2+6a+1$

17. Find the product. Use special product formulas to simplify: $(a+8)(a-8)$

$=a^2-64$

18. Find the product. Use special product formulas to simplify: $(m+2)^2$

$=m^2+2\cdot m\cdot 2+2^2$

$=m^2+4m+4$

19. Find the product. Use special product formulas to simplify: $(5y+3x)(5y-3x)$

$=25y^2-9x^2$

20. Find the product. Use special product formulas to simplify: $(z-5)^2$

$=z^2-2\cdot z\cdot 5+5^2$ $\quad\quad =z^2-10z+25$

21. Find the product. Use special product formulas to simplify: $(2r+5t)^2$

$=(2r)^2+2\cdot 2r\cdot 5t+(5t)^2 \qquad =4r^2+20rt+25t^2$

22. Find the product. Use special product formulas to simplify: $(6m-5)(6m+5)$

$=(6m)^2-(5)^2 \qquad =36m^2-25$

23. Find the product. Use special product formulas to simplify: $p(3p+7)(3p-7)$

$=p[(3p)^2-(7)^2] \qquad = p \;\; 9p^2-49 \qquad =9p^3-49p$

24. Find the product. Use special product formulas to simplify: $\left(\frac{3}{4}-x\right)\left(\frac{3}{4}+x\right)$

$=\left[\left(\frac{3}{4}\right)^2-(x)^2\right] \qquad =\frac{9}{16}-x^2$

25. Find the product. Use special product formulas to simplify: $-(4r-2)^2$

$=-[(4r)^2-2\cdot 4r\cdot 2+(2)^2] \qquad =-(16r^2-16r+4) \qquad =-16r^2+16r-4$

26. Perform the division: $\dfrac{8t^5-4t^3+4t^2}{2t}$

$=\frac{8t^5}{2t}-\frac{4t^3}{2t}+\frac{4t^2}{2t} \qquad =4t^4-2t^2+2t$

27. Perform the division: $\dfrac{20m^5-10m^4+5m^2}{5m^2}$

$=\frac{20m^5}{5m^2}-\frac{10m^4}{5m^2}+\frac{5m^2}{5m^2} \qquad =4m^3-2m^2+1 \qquad =25y^2-9x^2$

28. Perform the division: $(-10m^4n^2+5m^3n^2+6m^2n^4)\div 5m^2n$

$=\frac{-10m^4n^2}{5m^2n}+\frac{5m^3n^2}{5m^2n}+\frac{6m^2n^4}{5m^2n} =-2m^2n+mn+\frac{6}{5}n^3$

29. Perform the division: $\dfrac{2r^3-5r^2-6r+15}{r-3}$

$$2r^2 + r - 3 + \frac{6}{r-3}$$
$$r - 3 \overline{)\, 2r^3 - 5r^2 - 6r + 15}$$
$$\underline{-2r^3 + 6r^2}$$
$$r^2 - 6r + 15$$
$$\underline{-r^2 + 3r}$$
$$-3r + 15$$
$$\underline{+3r - 9}$$
$$+6$$

30. Perform the division: $\frac{16x^2 - 25}{4x + 5}$

$$4r - 5$$
$$4x + 5 \overline{)\, 16x^2 - 25}$$
$$\underline{-16x^2 - 20r}$$
$$-20r - 25$$
$$\underline{20r + 25}$$
$$0$$

31. Perform the division: $\frac{5 - 2r^2 + r^4}{r^2 - 1}$

$$r^2 - 1 + \frac{4}{r^2 - 1}$$
$$r^2 - 1 \overline{)\, r^4 - 2r^2 + 5}$$
$$\underline{-r^4 + r^2}$$
$$-r^2 + 5$$
$$\underline{+r^2 - 1}$$
$$+4$$

Chapter 6.

1. Factor completely: $8m^2n^3 + 24m^2n^2$

$GCF = 8m^2n^2$

$= 8m^2n^2 \cdot n + 8m^2n^2 \cdot 3 \qquad = 8m^2n^2(n + 3)$

2. Factor completely: $5m^2 + 15mp - 2mr - 6pr$

$\underbrace{5m^2+15mp}_{5m}\underbrace{-2mr-6pr}_{-2r}$ $\quad = 5m(m+3p)-2r(m+3p) \quad = (m+3p)(5m-2r)$

3. Factor completely: $16m^3-4m^2p^2-4mp+p^3$

$\underbrace{16m^3-4m^2p^2}_{4m^2}\underbrace{-4mp+p^3}_{-p}$ $\quad = 4m^2(4m-p^2)-p(4m-p^2)$

$=(4m-p^2)(4m^2-p)$

4. Factor completely: $36p^6q+45p^5q^4+81p^3q^2$

$GCF = 9p^3q$

$=9p^3q\cdot 4p^3+9p^3q\cdot 5p^2q^3+9p^3q\cdot 9q \quad = 9p^3q(4p^3+5p^2q^3+9q)$

5. Factor completely: $r^2+3ra+2a^2$

$=(r+a)(r+2a)$

6. Factor completely: $5y^2-5y-30$

$GCF = 5$

$=5(y^2-y-6) \quad = 5(y-3)(y-2)$

7. Factor completely: $m^3n-2m^2n^2-3mn^3$

$GCF = mn$

$=mn(m^2-2mn-3n^2) \quad = mn(m-3n)(m+n)$

8. Factor completely: $(a+b)x^2+(a+b)x-12(a+b)$

$GCF = (a+b)$

$=(a+b)(x^2+x-12) \quad = (a+b)(x+4)(x-3)$

9. Factor completely: $6x^2-17x+12$

$=6x^2-8x-9x+12 \quad = \underbrace{6x^2-8x}_{2x}\underbrace{-9x+12}_{-3} \quad = 2x(3x-4)-3(3x-4)$

$=(3x-4)(2x-3)$

10. Factor completely: $2t^2+13t-18$

prime

11. Factor completely: $12s^2+11st-5t^2$

$12s^2+15st-4st-5t^2 \quad \underbrace{12s^2+15st}_{3s}\underbrace{-4st-5t^2}_{-t}$

$=3s(4s+5t)-t(4s+5t) \quad = (4s+5t)(3s-t)$

12. Factor completely: $18+65x+7x^2$

$18+63x+2x+7x^2 \qquad \underbrace{18+63x}_{9}+\underbrace{2x+7x^2}_{x} \qquad =9(2+7x)+x(2+7x)$

$=(2+7x)(9+x)$

13. Factor completely: x^4-1

$=(x^2+1)(x^2-1^2) \qquad =(x^2+1)(x+1)(x-1)$

14. Factor completely: $32a^2-8$

$GCF=8$

$=8(4a^2-1) \qquad =8[(2a)^2-1^2] \qquad =8(2a+1)(2a-1)$

15. Factor completely: $x^2-10x+25$

$=x^2-2\cdot x\cdot 5+5^2 \qquad =(x-5)^2$

16. Factor completely: $2x^2+24x+72$

$GCF=2$

$=2(x^2+12x+36) \qquad =2(x^2+2\cdot x\cdot 6+6^2) \qquad =2(x+6)^2$

17. Factor completely: $27t^3-64s^6$

$=(3t)^3-(4s^2)^3 \qquad =(3t-4s^2)[(3t)^2+(3t)(4s^2)+(4s^2)^2]$

$=(3t-4s^2)(9t^2+12ts^2+16s^4)$

18. Factor completely: $125t^3+8s^6$

$=(5t)^3+(2s^2)^3 \qquad =(5t+2s^2)[(5t)^2-(5t)(2s^2)+(2s^2)^2]$

$=(5t+2s^2)(25t^2-10ts^2+4s^4)$

19. Factor completely: $16r^2-25a^2$

$=(4r)^2-(5a)^2 \qquad =(4r+5a)(4r-5a)$

20. Factor completely: $81w^2+16$

prime

21. Solve the equation: $(2x+7)(x^2+2x-3)=0$

$(2x+7)(x+3)(x-1)=0$

$2x+7=0 \quad 2x=-7 \quad x_1=-3\frac{1}{2}$

$2x+7=0 \quad 2x=-7 \quad x_1=-3\frac{1}{2}$

$x-1=0 \quad x_3=1$

$\left\{-3\frac{1}{2},-3,1\right\}$

22. Solve the equation: $x^2+(x+1)^2=(x+2)^2$

$x^2+x^2+2x+1=x^2+4x+4$ $\quad x^2-2x-3=0$ $\quad (x-3)(x+1)=0$

$x-3=0 \quad x_1=3$

$x+1=0 \quad x_2=-1$

$\{3,-1\}$

23. Solve the equation: $x^3=3x+2x^2$

$x^3-2x^2-3x=0$

$GCF=x$

$x(x^2-2x-3)=0 \qquad x(x+1)(x-3)=0$

$x_1=0$

$x+1=0 \quad x_2=-1$

$x-3=0 \quad x_3=3$

$\{-1,0,3\}$

24. Solve the equation: $16r^3-9r=0$

$GCF=r$

$r(16r^2-9)=0 \qquad r[(4r)^2-3^2]=0 \qquad r(4r-3)(4r+3)=0$

$r_1=0$

$4r-3=0 \quad r_2=\frac{3}{4}$

$4r+3=0 \quad r_3=-\frac{3}{4}$

$\left\{0,\frac{3}{4},-\frac{3}{4}\right\}$

25. A certain triangle has its base equal in measure to its height. The area of the triangle is $72\ m^2$. Find the base and height measure.

$let\ x\ is\ the\ base. The\ area\ of\ triangle\ is\ a=\frac{base\cdot height}{2}$

$a=\frac{x\cdot x}{2} \qquad \frac{x^2}{2}=72 \quad x^2=144 \quad x^2-144=0 \quad (x-12)(x+12)=0$

$x-12=0 \quad x_1=12$

$x+12=0 \quad x_2=-12$

$base\ is\ 12m: hegative\ solution\ does\ not\ make\ sense, since\ x\ represents\ lenght,$

$which\ cannot\ be\ negative.$

26. The product of the second and third of three consecutive integers is 2 more than10 times the first integer. Find the integers.

$Let\ the\ first\ conecutive\ integer\ is\ x, the\ second - x+1, and\ the\ third - x+2$

$(x+1)(x+2)=10x+2 \qquad x^2+2x+x+2=10x+2 \qquad x^2-7x=0$

$x\ \ x-7\ =0$

$x_1=0$

$x-7=0 \quad x_2=7$

$Three\ consecutive\ integer\ numbers\ are: 0,1,2\ or\ 7,8,9$

27. A ladder is leaning against a building. The distance from the bottom of the ladder to the building is 4 ft less than the length of the ladder. How high up the side of the building is the top of the ladder if that distance is 2 ft less than the length of the ladder.

$Let\ the\ length\ of\ the\ ladder\ is\ x$

$(x-4)^2+(x-2)^2=x^2 \qquad x^2-8x+16+x^2-4x+4=x^2$

$x^2-12x+20=0 \qquad (x-10)(x-2)=0$

$x-10=0 \quad x_1=10$

$x-2=0 \quad x_2=2$

$The\ length\ of\ the\ ladder\ is\ 10\,ft, and\ the\ high\ of\ the\ building\ is\ 8\,ft.$

$The\ solution\ 2\,ft\ give\ a\ negative\ value\ of\ distance\ from\ the\ bottom\ of\ the\ ladder$

$to\ the\ building, that\ does\ not\ make\ sence.$

28. An object projected from a height of 48 ft with an initial velocity of 32 $ft\ per\ sec$ after t seconds has height $h=-16t^2+32t+48$

(a) After how many seconds is the height 64 ft?

(b) After how many seconds does the object hit the ground?

$a)\ 64=-16t^2+32t+48 \qquad -16t^2+32t+48-64=0 \qquad -16t^2+32t-16=0$

$-16(t^2-2t+1)=0 \qquad t^2-2t+1=0 \qquad (t-1)^2=0$

$t-1=0 \quad t=1$

$Solution - 1\,sec$

$b)\ 0=-16t^2+32t+48$

$-16t^2+32t+48=0$ $\quad -16(t^2-2t-3)=0$ $\quad t^2-2t-3=0$

$(t-3)(t+1)=0$

$t-3=0 \quad t_1=3$

$t+1=0 \quad t_2=-1$

Solution is 3 *sec. The negative solution,* -1*, does not make sence, since t represents time, which cannot be negative.*

Chapter 7.

1. Write the rational expression in lowest terms: $\dfrac{7t^2-31t-20}{7t+4}$

$$=\frac{\overbrace{7t^2-35t}^{7t}\overbrace{+4t-20}^{4}}{7t+4} = \frac{7t(t-5)+4(t-5)}{7t+4} = \frac{(t-5)(7t+4)}{7t+4} = t-5$$

2. Write the rational expression in lowest terms: $\dfrac{x^2+2x-15}{x^2+6x+5}$

$$=\frac{(x+5)(x-3)}{(x+5)(x+1)} = \frac{x-3}{x+1}$$

3. Write the rational expression in lowest terms: $\dfrac{5k^2-13k-6}{5k+2}$

$$=\frac{\overbrace{5k^2-15k}^{5k}\overbrace{+2k-6}^{2}}{5k+2} = \frac{5k(k-3)+2(k-3)}{5k+2} = \frac{(k-3)(5k+2)}{5k+2} = k-3$$

4. Write the rational expression in lowest terms: $\dfrac{2x^2-3x-5}{2x^2-7x+5}$

$$=\frac{\overbrace{2x^2-5x}^{x}\overbrace{+2x-5}^{1}}{\underbrace{2x^2-5x}_{x}\underbrace{-2x+5}_{-1}} = \frac{x(2x-5)+1(2x-5)}{x(2x-5)-1(2x-5)} = \frac{(2x-5)(x+1)}{(2x-5)(x-1)} = \frac{x+1}{x-1}$$

5. Multiply: $\dfrac{3x^2-5x-2}{x-2}\cdot\dfrac{x-3}{x+1}$

$$=\frac{\overbrace{3x^2-6x}^{3x}\overbrace{+x-2}^{1}}{x-2}\cdot\frac{x-3}{x+1} = \frac{3x(x-2)+1(x-2)}{x-2}\cdot\frac{x-3}{x+1}$$

$$=\frac{(x-2)(3x+1)(x-3)}{(x-2)(x+1)} = \frac{(3x+1)(x-3)}{x+1}$$

6. Multiply: $\dfrac{2k^2+3k-2}{6k^2-7k+2}\cdot\dfrac{4k^2-5k+1}{k^2+k-2}$

$$=\frac{\overbrace{2k^2+4k}^{2k}\overbrace{-k-2}^{-1}}{\underbrace{6k^2-4k}_{2k}\underbrace{-3k+2}_{-1}}\cdot\frac{\overbrace{4k^2-4k}^{4k}\overbrace{-k+1}^{-1}}{(k+2)(k-1)}$$

$$=\frac{2k(k+2)-1(k+2)}{2k(3k-2)-1(3k-2)}\cdot\frac{4k(k-1)-1(k-1)}{(k+2)(k-1)}$$

$$=\frac{(k+2)(2k-1)(k-1)(4k-1)}{(3k-2)(2k-1)(k+2)(k-1)} \qquad =\frac{4k-1}{3k-2}$$

7. Divide: $\dfrac{m^2+2mp-3p^2}{m^2-3mp+2p^2}\div\dfrac{m^2+4mp+3p^2}{m^2+2mp-8p^2}$

$$=\frac{(m+3p)(m-p)}{(m-2p)(m-p)}\div\frac{(m+3p)(m+p)}{(m-2p)(m+4p)}$$

$$=\frac{(m+3p)(m-p)}{(m-2p)(m-p)}\cdot\frac{(m-2p)(m+4p)}{(m+3p)(m+p)} \qquad =\frac{m+4p}{m+p}$$

8. Divide: $\dfrac{(q-3)^4(q+2)^2}{q^2+3q+2}\div\dfrac{q^2-6q+9}{q^2+4q+4}$

$$=\frac{(q-3)^4(q+2)^2}{(q+2)(q+1)}\div\frac{(q-3)^2}{(q+2)^2} \qquad =\frac{(q-3)^4(q+2)^2}{(q+2)(q+1)}\cdot\frac{(q+2)^2}{(q-3)^2}$$

$$=\frac{(q-3)^2(q+2)^2}{(q+1)}$$

9. Add: $\dfrac{4m}{m^2+3m+2}+\dfrac{2m-1}{m^2+6m+5}$

$$=\frac{4m}{(m+2)(m+1)}+\frac{2m-1}{(m+5)(m+1)}$$

$$LCD=(m+2)(m+1)(m+5)$$

$$=\frac{4m(m+5)}{(m+2)(m+1)(m+5)}+\frac{(2m-1)(m+2)}{(m+2)(m+1)(m+5)}$$

$$=\frac{4m^2+20m}{(m+2)(m+1)(m+5)}+\frac{4m^2+4m-m-2}{(m+2)(m+1)(m+5)}$$

$$=\frac{4m^2+20m+4m^2+4m-m-2}{(m+2)(m+1)(m+5)} \qquad =\frac{8m^2+23m-2}{(m+2)(m+1)(m+5)}$$

10. Add: $\dfrac{a}{a^2+3a-4}+\dfrac{4a}{a^2+7a+12}$

$$=\frac{a}{(a+4)(a-1)}+\frac{4a}{(a+4)(a+3)}$$

$$LCD=(a+4)(a-1)(a+3)$$

$$=\frac{a(a+3)}{(a+4)(a-1)(a+3)}+\frac{4a(a-1)}{(a+4)(a-1)(a+3)}$$

$$=\frac{a^2+3a}{(a+4)(a-1)(a+3)}+\frac{4a^2-4a}{(a+4)(a-1)(a+3)} \qquad =\frac{a^2+3a+4a^2-4a}{(a+4)(a-1)(a+3)}$$

$$=\frac{5a^2-a}{(a+4)(a-1)(a+3)} \qquad =\frac{a(5a-1)}{(a+4)(a-1)(a+3)}$$

11. Perform indicated operation: $\frac{2x-z}{2x^2+xz-10z^2}-\frac{x+z}{x^2-4z^2}$

$$=\frac{2x-z}{\underbrace{2x^2+5xz}_{x}\underbrace{-4xz-10z^2}_{-2z}}-\frac{x+z}{x^2-(2z)^2}$$

$$=\frac{2x-z}{x(2x+5z)-2z(2x+5z)}-\frac{x+z}{(x-2z)(2x+5z)}$$

$$=\frac{2x-z}{(2x+5z)(x-2z)}-\frac{x+z}{(x-2z)(x+2z)}$$

$$LCD=(2x+5z)(x-2z)(x+2z)$$

$$=\frac{(2x-z)(x+2z)}{(2x+5z)(x-2z)(x+2z)}-\frac{(x+z)(2x+5z)}{(2x+5z)(x-2z)(x+2z)}$$

$$=\frac{2x^2+4xz-xz-2z^2}{(2x+5z)(x-2z)(x+2z)}-\frac{2x^2+5xz+2xz+5z^2}{(2x+5z)(x-2z)(x+2z)}$$

$$=\frac{2x^2+3xz-2z^2}{(2x+5z)(x-2z)(x+2z)}-\frac{2x^2+7xz+5z^2}{(2x+5z)(x-2z)(x+2z)}$$

$$=\frac{2x^2+3xz-2z^2-2x^2-7xz-5z^2}{(2x+5z)(x-2z)(x+2z)} \qquad =\frac{\overbrace{-4xz-7z^2}^{-z}}{(2x+5z)(x-2z)(x+2z)}$$

$$=\frac{-z(4x+7z)}{(2x+5z)(x-2z)(x+2z)}$$

12. Perform indicated operation: $\frac{6}{k^2+3k}-\frac{1}{k^2-k}+\frac{2}{k^2+2k-3}$

$$= \frac{6}{\underbrace{k^2+3k}_{k}} - \frac{1}{\underbrace{k^2-k}_{k}} + \frac{2}{(k+3)(k-1)} \qquad = \frac{6}{k(k+3)} - \frac{1}{k(k-1)} + \frac{2}{(k+3)(k-1)}$$

$$LCD = k(k+3)(k-1)$$

$$= \frac{6(k-1)}{k(k+3)(k-1)} - \frac{(k+3)}{k(k+3)(k-1)} + \frac{2k}{k(k+3)(k-1)}$$

$$= \frac{6k-6}{k(k+3)(k-1)} - \frac{k+3}{k(k+3)(k-1)} + \frac{2k}{k(k+3)(k-1)} \qquad = \frac{6k-6-k-3+2k}{k(k+3)(k-1)}$$

$$= \frac{7k-9}{k(k+3)(k-1)}$$

13. Simplify: $\dfrac{\frac{1}{x}+x}{\frac{x^2+1}{8}}$

$$LCD = 8x$$

$$= \frac{8x \cdot \frac{1}{x} + x \cdot 8x}{8x \cdot \frac{x^2+1}{8}} \qquad = \frac{8+8x^2}{x(x^2+1)} \qquad = \frac{8(x^2+1)}{x(x^2+1)} \qquad = \frac{8}{x}$$

14. Simplify: $\dfrac{\frac{1}{m+1}-1}{\frac{1}{m+1}+1}$

$$LCD = m+1$$

$$= \frac{(m+1)\frac{1}{m+1} - 1(m+1)}{(m+1)\frac{1}{m+1} + 1(m+1)} \qquad = \frac{1-m-1}{1+m+1} \qquad = -\frac{m}{m+2}$$

15. Simplify: $\dfrac{\frac{1}{m-1}+\frac{2}{m+2}}{\frac{2}{m+2}-\frac{1}{m-3}}$

$$LCD = (m-1)(m+2)(m-3)$$

$$= \frac{(m-1)(m+2)(m-3)\frac{1}{(m-1)} + \frac{2}{(m+2)}(m-1)(m+2)(m-3)}{(m-1)(m+2)(m-3)\frac{2}{(m+2)} - \frac{1}{(m-3)}(m-1)(m+2)(m-3)}$$

$$=\frac{(m+2)(m-3)+2(m-1)(m-3)}{2(m-1)(m-3)-(m-1)(m+2)} \qquad =\frac{m^2-m-6+2m^2-8m+6}{2m^2-8m+6-m^2-m+2}$$

$$=\frac{3m^2-9m}{m^2-9m+8} \qquad =\frac{3m(m-3)}{(m-1)(m-8)}$$

16. Simplify: $1+\cfrac{1}{1+\cfrac{1}{1+1}}$

$$=1+\cfrac{1}{1+\cfrac{1}{2}} \qquad =1+\cfrac{1}{\cfrac{3}{2}} \qquad =1+1\div\frac{3}{2} \qquad =1+1\cdot\frac{2}{3} \qquad =\frac{5}{3}$$

17. Solve the equation and check your solutions: $\frac{2p}{p^2-1}=\frac{2}{p+1}-\frac{1}{p-1}$

$$\frac{2p}{(p-1)(p+1)}=\frac{2}{p+1}-\frac{1}{p-1} \qquad \frac{2p}{(p-1)(p+1)}=\frac{2(p-1)}{(p-1)(p+1)}-\frac{1(p+1)}{(p-1)(p+1)}$$

$$\frac{2p}{(p-1)(p+1)}=\frac{2p-2}{(p-1)(p+1)}-\frac{p+1}{(p-1)(p+1)} \qquad \frac{2p}{(p-1)(p+1)}=\frac{2p-2-p-1}{(p-1)(p+1)}$$

$$\frac{2p}{(p-1)(p+1)}=\frac{p-3}{(p-1)(p+1)}$$

$$(p-1)(p+1)\frac{2p}{(p-1)(p+1)}=(p-1)(p+1)\frac{p-3}{(p-1)(p+1)}$$

$$2p=p-3 \qquad p=-3$$

$$Check: \frac{2(-3)}{(-3)^2-1}=\frac{2}{(-3)+1}-\frac{1}{(-3)-1}$$

$$\frac{-6}{9-1}=\frac{2}{-2}-\frac{1}{-4} \qquad -\frac{3}{4}=-1+\frac{1}{4} \qquad -\frac{3}{4}=-\frac{3}{4} \quad true$$

The solution set is $\{-3\}$

18. Solve the equation and check your solutions: $\frac{k}{k-4}-5=\frac{4}{k-4}$

$$\frac{k}{k-4}-\frac{5(k-4)}{k-4}=\frac{4}{k-4} \qquad \frac{k}{k-4}-\frac{5k-20}{k-4}=\frac{4}{k-4} \qquad \frac{k-5k+20}{k-4}=\frac{4}{k-4}$$

$$(k-4)\frac{-4k+20}{k-4}=(k-4)\frac{4}{k-4} \qquad -4k+20=4 \quad -4k=-16 \qquad k=4$$

$Check: \frac{4}{4-4} - 5 = \frac{4}{4-4}$

The proposed solution, $4,$ *makes an original denominator equal* $0,$
so is not a solution.

$\varnothing$

19. Solve the equation and check your solutions: $\frac{-2}{z+5} + \frac{3}{z-5} = \frac{20}{z^2-25}$

$\frac{-2}{z+5} + \frac{3}{z-5} = \frac{20}{z^2-5^2}$ $\qquad$ $\frac{-2(z-5)}{(z-5)(z+5)} + \frac{3(z+5)}{(z-5)(z+5)} = \frac{20}{(z-5)(z+5)}$

$\frac{-2z+10}{(z-5)(z+5)} + \frac{3z+15}{(z-5)(z+5)} = \frac{20}{(z-5)(z+5)}$

$\frac{-2z+10+3z+15}{(z-5)(z+5)} = \frac{20}{(z-5)(z+5)}$

$(z-5)(z+5)\frac{z+25}{(z-5)(z+5)} = (z-5)(z+5)\frac{20}{(z-5)(z+5)}$ $\qquad$ $z+25=20$

$z=-5$

$Check: \frac{-2}{-5+5} + \frac{3}{-5-5} = \frac{20}{(-5)^2-5^2}$

The proposed solution, $-5,$ *makes an original denominator equal* $0,$
so is not a solution.
The solution set is $\varnothing$

20. Solve the equation and check your solutions:

$\frac{x+4}{x^2-3x+2} - \frac{5}{x^2-4x+3} = \frac{x-4}{x^2-5x+6}$

$\frac{x+4}{(x-1)(x-2)} - \frac{5}{(x-1)(x-3)} = \frac{x-4}{(x-2)(x-3)}$

$\frac{(x+4)(x-3)}{(x-2)(x-1)(x-3)} - \frac{5(x-2)}{(x-2)(x-1)(x-3)} = \frac{(x-4)(x-1)}{(x-2)(x-1)(x-3)}$

$\frac{(x+4)(x-3)-5(x-2)}{(x-2)(x-1)(x-3)} = \frac{(x-4)(x-1)}{(x-2)(x-1)(x-3)}$

$\frac{x^2-3x+4x-12-5x+10}{(x-2)(x-1)(x-3)} = \frac{x^2-x-4x+4}{(x-2)(x-1)(x-3)}$

$(x-2)(x-1)(x-3)\frac{x^2-4x-2}{(x-2)(x-1)(x-3)} = (x-2)(x-1)(x-3)\frac{x^2-5x+4}{(x-2)(x-1)(x-3)}$

$x^2 - 4x - 2 = x^2 - 5x + 4 \qquad x = 6$

$$Check: \frac{6+4}{6^2 - 3\cdot 6 + 2} - \frac{5}{6^2 - 4\cdot 6 + 3} = \frac{6-4}{6^2 - 5\cdot 6 + 6}$$

$$\frac{10}{36-18+2} - \frac{5}{36-24+3} = \frac{2}{36-30+6}$$

$$\frac{10}{20} - \frac{5}{15} = \frac{2}{12}$$

$$\frac{1}{2} - \frac{1}{3} = \frac{1}{6} \quad true$$

The proposed solution, 6, *does not make an original denominator equal* 0,

so is a solution.

The solution set is $\{6\}$

21. One-third of a number is 2 more than one-sixth of the same number. What is the number?

Let the number is x

$$\frac{1}{3}x = \frac{1}{6}x + 2$$

$LCD = 6$

$$6\left(\frac{1}{3}x\right) = 6\left(\frac{1}{6}x + 2\right) \qquad 2x = x + 12 \qquad x = 12$$

The number is 12

22. A boat can go 20 mi against a current in the same time that it can go 60 mi with the current. The current is 4 mph. Find the speed of the boat in still water.

Let the speed of the boat in the still water is x

$$\frac{20}{x-4} = \frac{60}{x+4}$$

$LCD = (x+4)(x-4)$

$$(x+4)(x-4)\frac{20}{x-4} = (x+4)(x-4)\frac{60}{x+4} \qquad 20(x+4) = 60(x-4)$$

$$20x + 80 = 60x - 240 \qquad 40x = 320 \qquad x = 8$$

The speed of the boat in the still water is 8 *mph*

23. Working alone, Jorge can paint a room in 8 hr. Caterina can paint the same room working alone in 6 hr. How long will it take them if they work together?

$Let\ x = the\ number\ of\ hours\ it\ takes\ Jorge\ and\ Caterina\ to\ paint\ a\ room,$
$working\ together$

$$\frac{1}{8}x + \frac{1}{6}x = 1$$

$$LCD = 24$$

$$24\left(\frac{1}{8}x + \frac{1}{6}x\right) = 24 \cdot 1 \qquad 3x + 4x = 24 \qquad 3x = 24 \qquad x = \frac{24}{7} \quad x = 3\frac{3}{7}$$

$Working\ together,\ Jorge\ and\ Caterina\ can\ paint\ a\ room\ in\ 3\frac{3}{7}\ hr.$

$$20x + 80 = 60x - 240$$

24. One pipe can fill a swimming pool in 6 hr, and another pipe can do it in 9 hr. How long will it take the two pipes working together to fill the pool $\frac{3}{4}$ full?

$Let\ x = the\ number\ of\ hours\ it\ takes\ two\ pipes\ to\ fill\ a\ pool\ \frac{3}{4}\ full,$

$working\ together$

$$\frac{1}{6}x + \frac{1}{9}x = \frac{3}{4}$$

$$LCD = 36$$

$$36\left(\frac{1}{6}x + \frac{1}{9}x\right) = 36 \cdot \frac{3}{4} \qquad 6x + 4x = 27 \qquad 10x = 27 \qquad x = 2\frac{7}{10}$$

$Working\ together,\ two\ pipes\ can\ fill\ a\ pool\ \frac{3}{4}\ full\ in\ 2\frac{7}{10}\ hr.$

Chapter 8.

1. Find the square root: $\sqrt{3^2 + 4^2}$

$$= \sqrt{9+16} \qquad = \sqrt{25} \qquad = 5$$

2. Find the square root: $\sqrt{5^2 + 12^2}$

$$= \sqrt{25+144} \qquad = \sqrt{169} \qquad = 13$$

3. Find the distance between the pair of points: $(5,7)$ *and* $(1,4)$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad = \sqrt{(1-5)^2 + (4-7)^2} \qquad = \sqrt{(-4)^2 + (-3)^2}$$

$$= \sqrt{16+9} \qquad = \sqrt{25} \qquad = 5$$

4. Find the distance between the pair of points: $(-3,-6)$ *and* $(-4,0)$

$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $= \sqrt{[-4-(-3)]^2+[0-(-6)]^2}$

$= \sqrt{(-1)^2+(6)^2}$ $= \sqrt{1+36}$ $= \sqrt{37}$

5. Simplify: $\sqrt{81m^4n^2}$

$= \sqrt{9^2(m^2)^2n^2}$ $= 9m^2n$

6. Simplify: $\sqrt{\dfrac{x^4y^6}{169}}$

$= \sqrt{\dfrac{(x^2)^2(y^3)^2}{13^2}}$ $= \dfrac{x^2y^3}{13}$

7. Simplify: $\sqrt{\dfrac{w^8z^{10}}{400}}$

$= \sqrt{\dfrac{(w^4)^2(z^5)^2}{20^2}}$ $= \dfrac{w^4z^5}{20}$

8. Simplify: $\sqrt[3]{\dfrac{n^9}{27}}$

$= \sqrt[3]{\dfrac{(n^3)^3}{3^3}}$ $= \dfrac{n^3}{3}$

9. Perform indicated operation: $3\sqrt{8x^2}-4x\sqrt{2}$

$= 3\sqrt{2\cdot 2^2x^2}-4x\sqrt{2}$ $= 3\cdot 2x\sqrt{2}-4x\sqrt{2}$ $= 6x\sqrt{2}-4x\sqrt{2}$ $= 2x\sqrt{2}$

10. Simplify and combine terms where possible: $3\sqrt{75}+2\sqrt{27}$

$= 3\sqrt{3\cdot 25}+2\sqrt{3\cdot 9}$ $= 3\sqrt{3\cdot 5^2}+2\sqrt{3\cdot 3^2}$ $= 3\cdot 5\sqrt{3}+2\cdot 3\sqrt{3}$

$= 15\sqrt{3}+6\sqrt{3}$ $= 21\sqrt{3}$

11. Simplify and combine terms where possible: $4\sqrt{24}-3\sqrt{54}+\sqrt{6}$

$= 4\sqrt{6\cdot 4}-3\sqrt{6\cdot 9}+\sqrt{6}$ $= 4\sqrt{6\cdot 2^2}-3\sqrt{6\cdot 3^2}+\sqrt{6}$

$= 4\cdot 2\sqrt{6}-3\cdot 3\sqrt{6}+\sqrt{6}$ $= 8\sqrt{3}-9\sqrt{3}+\sqrt{6}$ $= 0$

12. Simplify and combine terms where possible: $\sqrt{20m^2}-m\sqrt{45}$

$= \sqrt{5\cdot 4\cdot m^2}-m\sqrt{9\cdot 5}$ $= \sqrt{5\cdot 2^2\cdot m^2}-m\sqrt{5\cdot 3^2}$

$= 2m\sqrt{5} - 3m\sqrt{5} \qquad = -m\sqrt{5} \qquad = 0$

13. Simplify and combine terms where possible: $3k\sqrt{8k^2n} + 5k^2\sqrt{2n}$

$= 3k\sqrt{2\cdot 4\cdot k^2n} + 5k^2\sqrt{2n} \qquad = 3k\sqrt{2\cdot 2^2\cdot k^2n} + 5k^2\sqrt{2n}$

$= 3k\cdot 2k\sqrt{2n} + 5k^2\sqrt{2n} \qquad = 6k^2\sqrt{2n} + 5k^2\sqrt{2n} \quad = 11k^2\sqrt{2n}$

14. Perform indicated operation: $\sqrt{(-3-6)^2 + (2-4)^2}$

$= \sqrt{(-9)^2 + (-2)^2} \qquad = \sqrt{81+4} \qquad = \sqrt{85}$

15. Simplify the radical: $\sqrt{x^{10}y^{16}}$

$= \sqrt{(x^5)^2(y^8)^2} \qquad = x^5y^8$

16. Simplify the radical: $\sqrt{a^{15}b^{21}}$

$= \sqrt{(a^7)^2a(b^{10})^2b} \qquad = a^7b^{10}\sqrt{ab}$

17. Simplify the radical: $\sqrt{121x^6y^{10}}$

$= \sqrt{11^2(x^3)^2(y^5)^2} \qquad = 11x^3y^5$

18. Simplify the radical: $\sqrt{\dfrac{m^2n}{2}}$

$= \dfrac{m\sqrt{n}\sqrt{2}}{\sqrt{2}\sqrt{2}} \qquad = \dfrac{m\sqrt{2n}}{2}$

19. Simplify the radical: $\sqrt{\dfrac{2x^2z^4}{3y}}$

$= \dfrac{\sqrt{2x^2z^4}\sqrt{3y}}{\sqrt{3y}\sqrt{3y}} \qquad = \dfrac{xz^2\sqrt{6y}}{3y}$

20. Rationalize the denominator: $\dfrac{7}{2-\sqrt{11}}$

$= \dfrac{7(2+\sqrt{11})}{(2-\sqrt{11})(2+\sqrt{11})} \qquad = \dfrac{7(2+\sqrt{11})}{2^2-(\sqrt{11})^2} \qquad = \dfrac{7(2+\sqrt{11})}{4-11}$

$= \dfrac{7(2+\sqrt{11})}{-7} \qquad = -(2+\sqrt{11})$

21. Rationalize the denominator: $\dfrac{3+\sqrt{2}}{\sqrt{2}+1}$

$$=\frac{(3+\sqrt{2})(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} \quad =\frac{3\sqrt{2}-3+2-\sqrt{2}}{(\sqrt{2})^2-(1)^2} \quad =\frac{2\sqrt{2}-1}{2-1} \quad =2\sqrt{2}-1$$

22. Rationalize the denominator: $\dfrac{8}{4-\sqrt{x}}$

$$=\frac{8(4+\sqrt{x})}{(4-\sqrt{x})(4+\sqrt{x})} \quad =\frac{8(4+\sqrt{x})}{(4)^2-(\sqrt{x})^2} \quad =\frac{8(4+\sqrt{x})}{16-x}$$

23. Rationalize the denominator: $\dfrac{1}{\sqrt{x}+\sqrt{y}}$

$$=\frac{1(\sqrt{x}-\sqrt{y})}{(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})} \quad =\frac{\sqrt{x}-\sqrt{y}}{(\sqrt{x})^2-(\sqrt{y})^2} \quad =\frac{\sqrt{x}-\sqrt{y}}{x-y}$$

24. Solve the equation: $\sqrt{x+2}=3$

$$\sqrt{x+2}^{\;2}=3^2 \qquad x+2=9 \qquad x=7$$

$$Check: \sqrt{7+2}=3 \qquad \sqrt{9}=3 \qquad 3=3$$

The solution set is $\{7\}$

25. Solve the equation: $\sqrt{5x+11}=x+3$

$$\sqrt{5x+11}=x+3 \qquad (\sqrt{5x+11})^2=(x+3)^2 \qquad 5x+11=x^2+6x+9$$

$$x^2+x-2=0 \qquad (x+2)(x-1)=0$$

$$x+2=0 \quad x_1=-2$$

$$x-1=0 \quad x_2=1$$

$$Check: if\ x_1=-2 \qquad \sqrt{5(-2)+11}=-2+3 \quad \sqrt{1}=1 \quad 1=1 \quad true$$

$$if\ x_2=1 \qquad \sqrt{5(1)+11}=1+3 \qquad \sqrt{16}=4 \qquad 4=4 \quad true$$

The solution set is $-2,1$

26. Solve the equation: $\sqrt{3x+3}+\sqrt{x+2}=5$

$$\sqrt{3x+3}+\sqrt{x+2}=5 \qquad (\sqrt{3x+3}+\sqrt{x+2})^2=(5)^2$$

$$(\sqrt{3x+3})^2+2\sqrt{3x+3}\sqrt{x+2}+(\sqrt{x+2})^2=25$$

$$3x+3+x+2+2\sqrt{3x+3}\sqrt{x+2}=25 \qquad 4x-20=-2\sqrt{3x+3}\sqrt{x+2}$$

$$[2(x-5)]^2=(-\sqrt{3x+3}\sqrt{x+2})^2 \qquad 4(x^2-10x+25)=(3x+3)(x+2)$$

$4x^2 - 40x + 100 = 3x^2 + 6x + 3x + 6 \quad x^2 - 49x + 94 = 0 \quad (x-47)(x-2) = 0$

$x - 47 = 0 \quad x_1 = 47$

$x - 2 = 0 \quad x_2 = 2$

Check :

$if\ x_1 = 47 \quad \sqrt{3 \cdot 47 + 3} + \sqrt{47 + 2} = 5 \quad \sqrt{141 + 3} + \sqrt{47 + 2} = 5$

$\sqrt{144} + \sqrt{49} = 5 \quad 12 + 7 = 5 \quad 19 = 5 \quad false$

$if\ x_2 = 2 \quad \sqrt{3 \cdot 2 + 3} + \sqrt{2 + 2} = 5 \quad \sqrt{9} + \sqrt{4} = 5$

$3 + 2 = 5 \quad 5 = 5 \quad true$

The solution set is $\{2\}$

27. Solve the equation: $\sqrt[3]{2x} = \sqrt[3]{5x + 2}$

$(\sqrt[3]{2x})^3 = (\sqrt[3]{5x + 2})^3 \quad 2x = 5x + 2 \quad 3x = -2 \quad x = -\frac{2}{3}$

$Check: if\ x = -\frac{2}{3} \quad \sqrt[3]{2\left(-\frac{2}{3}\right)} = \sqrt[3]{5\left(-\frac{2}{3}\right) + 2} \quad \sqrt[3]{-\frac{4}{3}} = \sqrt[3]{-\frac{10}{3} + 2}$

$\sqrt[3]{-\frac{4}{3}} = \sqrt[3]{-\frac{10}{3} + \frac{6}{3}} \quad \sqrt[3]{-\frac{4}{3}} = \sqrt[3]{-\frac{4}{3}} \quad true$

The solution set is $\left\{-\frac{2}{3}\right\}$

28. Simplify: $x^{2/5} \cdot x^{7/5}$

$= x^{2/5 + 7/5} \quad = x^{9/5}$

29. Simplify: $(p^4 q^{1/2})^{4/3}$

$= p^{4 \bullet 4/3} q^{1/2 \bullet 4/3} \quad = p^{16/3} q^{2/3}$

30. Simplify: $\frac{q^{5/6} \cdot q^{-1/6}}{q^{1/3}}$

$= q^{5/6 + (-1/6) - 1/3} \quad = q^{1/3}$

31. Simplify: $(m^3 n^{1/4})^{2/3}$

$= m^{3 \bullet 2/3} n^{1/4 \bullet 2/3} \quad = m^2 n^{1/6}$

Chapter 9.

1. Solve the equation by using square root property: $(5z+6)^2 = 75$

$\sqrt{(5z+6)^2} = \sqrt{75}$

$5z+6 = 5\sqrt{3}$ *or* $5z+6 = -5\sqrt{3}$

$5z = -6+5\sqrt{3}$ *or* $5z = -6-5\sqrt{3}$

$z = \dfrac{-6+5\sqrt{3}}{5}$ *or* $z = \dfrac{-6-5\sqrt{3}}{5}$

The solution set is $\left\{\dfrac{-6\pm 5\sqrt{3}}{5}\right\}$

2. Solve the equation by using square root property: $(4x-3)^2 = 9$

$\sqrt{(4x-3)^2} = \sqrt{9}$

$4x-3 = 3$ *or* $4x-3 = -3$

$4x = 6$ *or* $4x = 0$

$x = \dfrac{3}{2}$ *or* $x = 0$

The solution set is $\left\{0, \dfrac{3}{2}\right\}$

3. Solve the equation by using square root property: $(4k-1)^2 - 48 = 0$

$(4k-1)^2 = 48$

$\sqrt{(4k-1)^2} = \sqrt{16\cdot 3}$

$4k-1 = 4\sqrt{3}$ *or* $4k-1 = -4\sqrt{3}$

$4k = 1+4\sqrt{3}$ *or* $4k = 1-4\sqrt{3}$

$k = \dfrac{1+4\sqrt{3}}{4}$ *or* $k = \dfrac{1-4\sqrt{3}}{4}$

The solution set is $\left\{\dfrac{1\pm 4\sqrt{3}}{4}\right\}$

4. Solve the equation by using square root property: $(m+2)^2 = 17$

$\sqrt{(m+2)^2} = \sqrt{17}$

$m+2=\sqrt{17}$ *or* $m+2=-\sqrt{17}$

$m=-2+\sqrt{17}$ *or* $m=-2-\sqrt{17}$

the solution set is $\{-2\pm\sqrt{17}\}$

5. Solve the equation by completing the square: $4x^2+4x=3$

$(2x)^2+2\cdot 2x\cdot 1+1-1=3$ $\quad (2x+1)^2=4$ $\quad \sqrt{(2x+1)^2}=\sqrt{4}$

$2x+1=2$ *or* $2x+1=-2$

$2x=1$ *or* $2x=-3$

$x=\frac{1}{2}$ *or* $x=-\frac{3}{2}$

The solution set is $\left\{-\frac{3}{2},\frac{1}{2}\right\}$

6. Remove parentheses and solve the equation by completing the square:

$(r-3)(r-5)=2$ $\quad r^2-5r-3r+15=2$ $\quad r^2-8r+15=2$

$r^2-2r\cdot 4+16-1=2$ $\quad (r-4)^2=3$ $\quad \sqrt{(r-4)^2}=\sqrt{3}$

$r-4=\sqrt{3}$ *or* $r-4=-\sqrt{3}$

$r=4+\sqrt{3}$ *or* $r=4-\sqrt{3}$

The solution set is $\{4\pm\sqrt{3}\}$

7. Solve the equation by completing the square: $3k^2+7k=4$

$\frac{3k^2+7k}{3}=\frac{4}{3}$ $\quad k^2+\frac{7k}{3}=\frac{4}{3}$ $\quad k^2+2\cdot k\cdot\frac{7}{6}+\left(\frac{7}{6}\right)^2-\left(\frac{7}{6}\right)^2=\frac{4}{3}$

$k^2+2\cdot k\cdot\frac{7}{6}+\left(\frac{7}{6}\right)^2=\frac{4}{3}+\left(\frac{7}{6}\right)^2$ $\quad \left(k+\frac{7}{6}\right)^2=\frac{4}{3}+\frac{49}{36}$ $\quad \sqrt{\left(k+\frac{7}{6}\right)^2}=\sqrt{\frac{97}{36}}$

$k+\frac{7}{6}=\frac{\sqrt{97}}{6}$ *or* $k+\frac{7}{6}=-\frac{\sqrt{97}}{6}$

$k=-\frac{7}{6}+\frac{\sqrt{97}}{6}$ *or* $k=-\frac{7}{6}-\frac{\sqrt{97}}{6}$

$k=\frac{-7+\sqrt{97}}{6}$ *or* $k=\frac{-7-\sqrt{97}}{6}$

The solution set is $\left\{\frac{-7\pm\sqrt{97}}{6}\right\}$

8. Solve the equation by completing the square: $(k-1)(k-7)=1$

$k^2-7k-k+7=1 \qquad k^2-8k+7=1 \qquad k^2-2\cdot k\cdot 4+16-9=1$

$(k-4)^2=10 \qquad \sqrt{(k-4)^2}=\sqrt{10}$

$k-4=\sqrt{10} \quad or \quad k-4=-\sqrt{10}$

$k=4+\sqrt{10} \quad or \quad k=4-\sqrt{10}$

The solution set is $\{4\pm\sqrt{10}\}$

9. Use the quadratic formula to solve the equation: $r^2-8r-9=0$

$$r=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad a=1, b=-8, c=-9$$

$$r=\frac{-(-8)\pm\sqrt{(-8)^2-4\cdot 1\cdot(-9)}}{2\cdot 1} \qquad r=\frac{8\pm\sqrt{64+36}}{2} \qquad r=\frac{8\pm 10}{2}$$

The solution set is $\{-1, 9\}$

10. Use the quadratic formula to solve the equation: $(2x+1)(x+1)=7$

$2x^2+2x+x+1=7 \qquad 2x^2+3x-6=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad a=2, b=3, c=-6$$

$$x=\frac{-3\pm\sqrt{3^2-4\cdot 2\cdot(-6)}}{2\cdot 2} \qquad x=\frac{-3\pm\sqrt{9+48}}{2} \qquad x=\frac{-3\pm\sqrt{57}}{2}$$

The solution set is $\left\{\frac{-3\pm\sqrt{57}}{4}\right\}$

11. Use the quadratic formula to solve the equation: $2x^2+x-5=0$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad a=2, b=1, c=-5$$

$$x=\frac{-1\pm\sqrt{1^2-4\cdot 2\cdot(-5)}}{2\cdot 2} \qquad x=\frac{-1\pm\sqrt{1+40}}{4} \qquad x=\frac{-1\pm\sqrt{41}}{4}$$

$$x = -\frac{1}{4} \pm \frac{\sqrt{41}}{4}$$

The solution set is $\left\{ -\frac{1}{4} \pm \frac{\sqrt{41}}{4} \right\}$

12. Use the quadratic formula to solve the equation: $4x^2 - x + 4 = x + 7$

$$4x^2 - 2x - 3 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad a = 4, b = -2, c = -3$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 4 \cdot (-3)}}{2 \cdot 4} \qquad x = \frac{2 \pm \sqrt{4 + 48}}{8} \qquad x = \frac{2 \pm \sqrt{4 \cdot 13}}{8}$$

$$x = \frac{2 \pm 2\sqrt{13}}{8} \quad x = \frac{1 \pm \sqrt{13}}{4}$$

The solution set is $\left\{ \frac{1 \pm \sqrt{13}}{4} \right\}$

13. A farmer has a rectangular cattle pen with perimeter 350 ft and area 7500 ft^2. What are the dimensions of the pen?

let length is x and width is y

$$2(x + y) = 350 \qquad xy = 7500 \qquad 2x + 2y = 350 \quad 2x = 350 - 2y \quad x = 175 - y$$

$$(175 - y)y = 7500 \qquad -y^2 + 175y = 7500 \qquad y^2 - 175y + 7500 = 0$$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad a = 1, b = -175, c = 7500$$

$$y = \frac{-(-175) \pm \sqrt{(-175)^2 - 4(-7500)}}{2} \qquad y = \frac{175 \pm \sqrt{30625 - 30000}}{2}$$

$$y = \frac{175 \pm \sqrt{625}}{2} \qquad y = \frac{175 \pm 25}{2}$$

$$y_1 = \frac{175 + 25}{2} \quad y_1 = \frac{200}{2} \quad y_1 = 100 \quad x_1 = 75$$

$$y_2 = \frac{175 - 25}{2} \quad y_2 = \frac{150}{2} \quad y_2 = 75 \quad x_2 = 100$$

dimensions of the pen are 75 *ft by* 100 *ft*

14. The base of a triangle measures 1 m more then three times the height of the triangle. The area of the triangle is $15m^2$. Find the lengths of the base and the height.

$$let\ height\ is\ x, the\ base\ is\ 3x+1$$

$$\frac{x(3x+1)}{2}=15 \qquad 3x^2+x=30 \qquad 3x^2+x-30=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad a=3, b=1, c=-30$$

$$x=\frac{-(1)\pm\sqrt{(1)^2-4\cdot3\cdot(-30)}}{2\cdot3} \qquad x=\frac{-1\pm\sqrt{361}}{2\cdot3} \qquad x=\frac{-1\pm19}{6}$$

$$x_1=\frac{-1+19}{6} \quad x_1=3$$

$$x_2=\frac{-1-19}{6} \quad x_2=-\frac{10}{3}$$

$$x_2=-\frac{10}{3}\ is\ not\ a\ solution, the\ side\ of\ triangle\ cannot\ be\ a\ negative$$

$$the\ height\ is\ 3, the\ base\ is\ 10$$

15. If an object is projected vertically into the air from ground level on Earth with an initial velocity of 64 $ft\ per\ sec$. Its altitude (height) s in feet after t seconds is given by the formula: $s=-16t^2+64t$.

At what time will the object be at a height of 64 ft?

$$64=-16t^2+64t \qquad 16t^2-64t+64=0 \qquad t^2-4t+4=0$$

$$t=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad a=1, b=-4, c=4$$

$$t=\frac{-(-4)\pm\sqrt{(-4)^2-4(4)}}{2} \qquad t=\frac{4\pm\sqrt{0}}{2} \quad t=\frac{4}{2} \quad t=2$$

$$solution \quad t=2\sec$$

16. In a right triangle, the lengths of the sides are consecutive integers. Use the Pythagorean formula to find these lengths.

$$let\ length\ of\ the\ first\ side\ is\ x, the\ second\ is\ x+1, and\ the\ third\ is\ x+2$$

$$x^2+(x+1)^2=(x+2)^2 \qquad x^2+x^2+2x+1=x^2+4x+4 \qquad x^2-2x-3=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad a=1, b=-2, c=-3$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-3)}}{2} \quad t = \frac{2 \pm \sqrt{4+12}}{2} \quad t = \frac{2 \pm \sqrt{16}}{2} \quad t = \frac{2 \pm 4}{2}$$

$t_1 = 3 \quad t_2 = -1$

the length of the first side cannot be negative

solution is $3, 4, 5$